Topology

B. Math. II

Back-paper Examination

Instructions: All questions carry equal marks.

1. Let X and Y be spaces and $A \subset X$ be a subspace. Let $f : A \to Y$ be a continuous map. If Y is Hausdorff, then show that there exists at most one extension of f to the closure $\overline{A}$. Give an example to show that this statement is false if Y is not Hausdorff. Justify your answer.

2. Let $\{X_\alpha\}$ be a collection of spaces and let $A_\alpha \subset X_\alpha$. If $\prod X_\alpha$ is given product topology, prove that
$$\prod \bar{A}_\alpha = \overline{\prod A_\alpha}$$

3. Define connected space. Let $f : S^1 \to \mathbb{R}$ be a continuous map, where S^1 denotes the unit circle in plane with subspace topology. Show that there exists a point $x \in S^1$ such that $f(x) = f(-x)$.

4. Define path connected space. Prove that if U is an open connected subspace of $\mathbb{R}^n$, then U is path connected.

5. Let X and Y be topological spaces and $\mathcal{A}$ be a collection of basic open sets in $X \times Y$ such that no finite subcollection of $\mathcal{A}$ covers $X \times Y$. If X is compact, show that there exists a point $x \in X$ such that no finite subcollection of $\mathcal{A}$ covers $\{x\} \times Y$.

6. State the *second countability axiom* for topological spaces. Prove that a metric space with a countable dense subset is second countable.

7. Define a regular topological space. Prove that if a space X is regular, then any two distinct points of X have open neighbourhoods whose closures are disjoint.

8. State Urysohn Lemma, Urysohn metrization theorem, Tietze's extension theorem and Tychonoff's theorem.